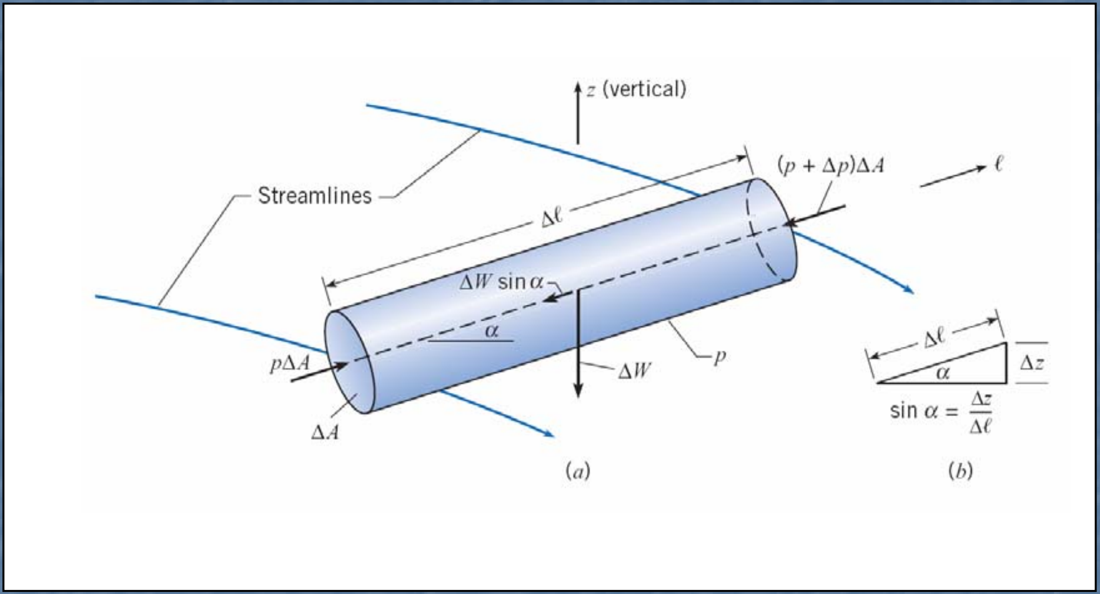
Fluid Motion

Euler Equation



Consider an element of fluid between two streamlines as shown

Applying Newton 2nd Law along the element, we have,

$$\sum F_l = ma_l$$

$$F_p + F_g = ma_l$$

Fluid Motion

$$\sum F_l = ma_l = p\Delta A - (p + \Delta p)\Delta A - \Delta W \sin\alpha$$

$$\rho V a_l = p\Delta A - p\Delta A - \Delta p \Delta A - \rho V g \sin\alpha$$

Re-Arranging

$$\rho(\Delta A \Delta l)a_l = p\Delta A - p\Delta A - \Delta p \Delta A - \rho(\Delta A \Delta l) g \sin\alpha$$

$$\rho(\Delta l)a_l = -\Delta p - \rho(\Delta l) g \sin\alpha$$

$$\rho a_l = -\frac{\Delta p}{\Delta l} - \rho g \sin\alpha$$

$\sin\alpha = \frac{dz}{dl}$ $(\gamma = \rho g)$ $\Delta l \to 0$

$$-\frac{dp}{dl} - \gamma \frac{dz}{dl} = \rho a_l$$

Euler Equation of motion

$$-\frac{d}{dl}(p + \gamma z) = \rho a_l$$

Problem 4.27 (p. 129)

Euler Equation

$$-\frac{d}{dl}(p+\gamma z)=\rho a_l$$

PROBLEM 4.27

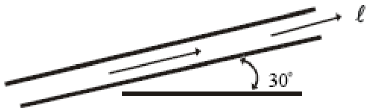
Situation: Flow through an inclined pipe at 30° from horizontal and decelerating at 0.3g.

Find: Pressure gradient in flow direction.

APPROACH

Apply Euler's equation.

ANALYSIS



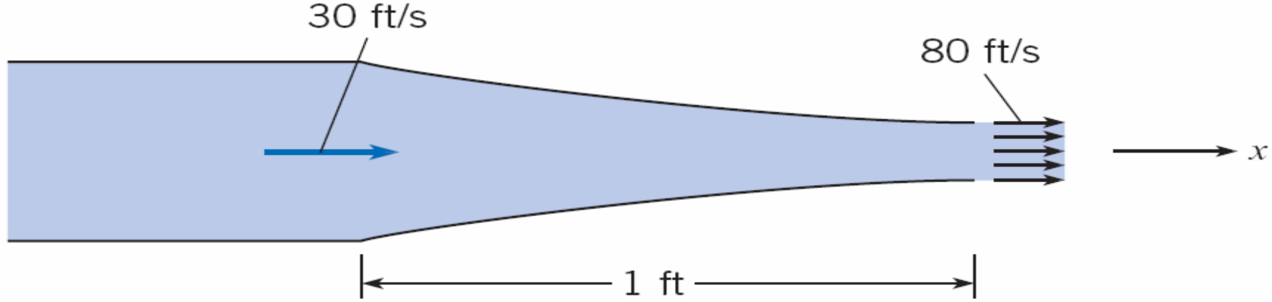
Euler's equation

$$\begin{aligned}
\partial/\partial\ell(p+\gamma z) &= -\rho a_\ell \\
\partial p/\partial\ell+\gamma\partial z/\partial\ell &= -\rho a_\ell \\
\partial p/\partial\ell &= -\rho a_\ell-\gamma\partial z/\partial\ell \\
&= -(\gamma/g)\times(-0.30g)-\gamma\sin 30^\circ \\
&= \gamma(0.30-0.50) \\
&\boxed{\partial p/\partial\ell=-0.20\gamma}
\end{aligned}$$

Problem 4.36 (p. 129)

$$a_t = \left(V \frac{\partial V}{\partial s} + \frac{\partial V}{\partial t} \right)$$

Find pressure gradient Half-way through the nozzle



APPROACH

Apply Euler's equation.

ANALYSIS

Euler's equation

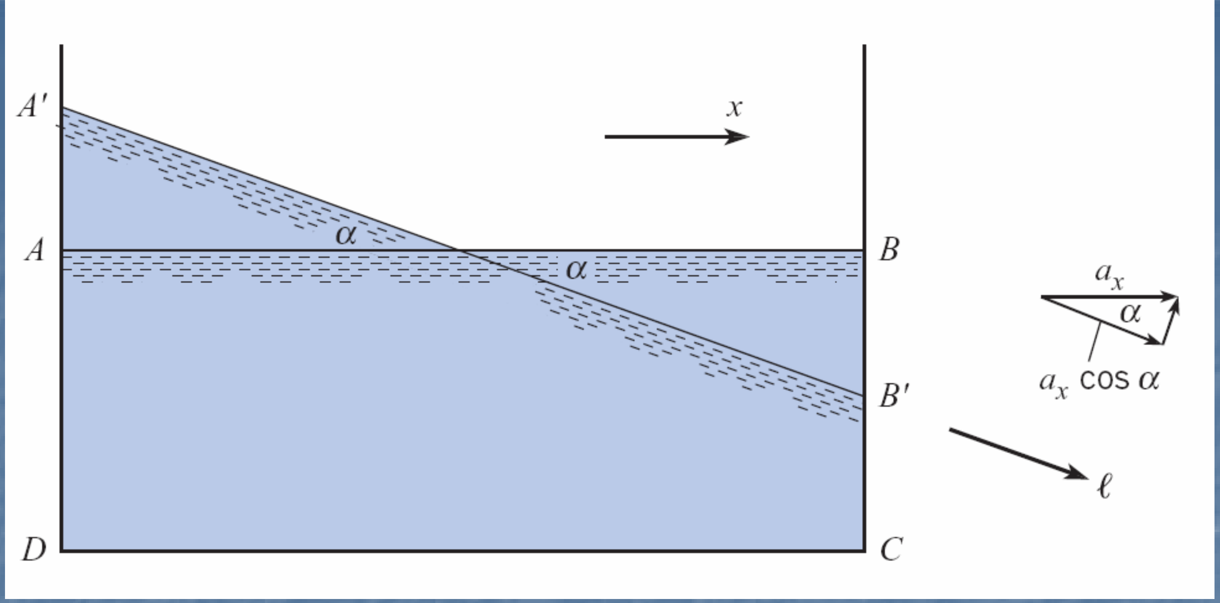
$$d/dx(p + \gamma z) = -\rho a_x$$

but z =const.; therefore

$$dp/dx = -\rho a_x$$
$$a_x = a_{\text{convective}} = V dV/dx$$
$$dV/dx = (80 - 30)/1 = 50 \text{ s}^{-1}$$
$$V_{\text{mid}} = (80 + 30)/2 = 55 \text{ ft/s}$$
$$= (55 \text{ ft/s})(50 \text{ ft/s/ft}) = 2,750 \text{ ft/s}^2$$

Finally

$$dp/dx = (-1.94 \text{ slug/ft}^3)(2,750 \text{ ft/s}^2)$$
$$\boxed{dp/dx = -5,335 \text{ psf/ft}}$$



Applying Euler formula along $A'B'$

$$-\frac{d}{dl}(p+\gamma z)=\rho a_l$$

$$\frac{dp}{dl}=0 \text{ as the change in pressure is zero}$$

$$-\frac{d}{dl}(\gamma z)=\rho a_l$$

$$\frac{dz}{dl}=\frac{a_l}{g}=\frac{a_x\cos\alpha}{g}$$

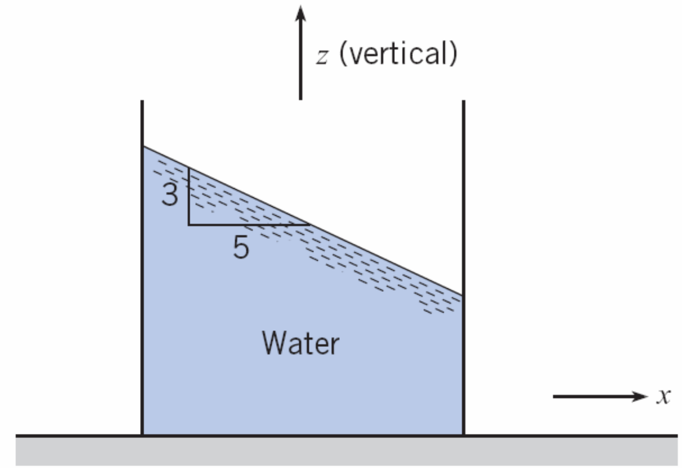
Problem 4.37 (p. 130)

$$-\frac{d}{dl}(p+\gamma z) = \rho a_l$$

$$\frac{dp}{dl} = 0 \textit{ as the change in pressure is zero}$$

$$-\frac{d}{dl}(\gamma z) = \rho a_l$$

$$\frac{dz}{dl} = \frac{a_l}{g} = \frac{a_x \cos\alpha}{g}$$



PROBLEM 4.37

Situation: Tank accelerated in x-direction to maintain liquid surface slope at -5/3.

Find: Acceleration of tank.

APPROACH

Apply Euler's equation.

ANALYSIS

Euler's equation. The slope of a free surface in an accelerated tank.

$$\tan\alpha = a_x/g$$

$$a_x = g\tan\alpha$$

$$= 9.81 \times 3/5$$

$$\boxed{a_x = 5.89 \text{ m/s}^2}$$

END OF LECTURE (4)